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# Review of theories regarding material bending

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**Abstract**—In the three roller bending machine, the three rollers rotate.Bending can be done in both sheet metal and bars of metal. For designing a three roller bending machine, it is required to calculate the exact force for bending. Based on this force, the machine parameters and motor power are decided.Various factors that should be considered while calculating this force are material properties, width, thickness, number of passes, bending radius, force developing mechanism and link.To analyse the force and power for motor the designer takes the help of analysis software. The cost of software for analysis is high. So there is requirement to find simple formula. In this paper the various theories regarding bending are reviewed, formulae for force and power calculation are collected and finally a case study is taken where we have put together all the results of these formulae.

**Index Terms**— bending force, bend radius, material thickness and width, Number of passes.

## 1 Introduction

Bending is a manufacturing process that produces a V-shape, U-shape, or channel shape etc. along a straight axis in ductile materials, most commonly sheet metal. During operation of bending, the inside surface is getting compressed and outer surface of the material is in tension as well as the strain in the bent material also increases with decreasing the radius of curvature. There are various types of bending methods such that V- Bending, Edge Bending, Air bending, Bottoming, Three-point bending, Folding, Wiping, Roll bending or section bending etc. Roll bending machines produce a bend across the width of flat or preformed metal to achieve a curved configuration. The roller bending machine is also called as a section bending machine. Roll-Benders are self-contained machines normally consisting of a base, chassis, stand, transmission drive, electrical system, rollers which are capable of producing a bend across the width of flat or preformed material by means of one or more rollers and other tooling to achieve a predetermined configuration. There are different factors which affect on the force required for the bending of the rod, those factors are material factors, machine factors, operation factors. In the material factors the width, thickness, material properties like Young's modulus, poison's ratio, and linear strain should be considered. In the machine factors type of loading, the distance between the rollers should be considered. The friction between rollers and material, angle to bend the material and numbers of passes to get a final end profile are operational factors. This bending force affect on machine capacity, size of rollers, linkage used for bending, power of motor, hydraulic system for the machine, gearbox capacity etc.. So it is required to estimate the force while designing the machine component.


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## 2 Related Work

In [1] author describes the mechanical dynamic analysis of steel sheet bending the three roller machine was performed by using the ABACUS/explicit code, Finite element simulation for three rollers bending machine.Two-dimensional FEM of this process built under ABAQUS. Author considers material properties definitions, curvature radius, varying distance between bottom two rollers and position of top rollers, the maps are generated. Due to that rolling process becomes easier and less time consuming. To validate the numerical model the industrial experiments using optimized numerical results carried out. Author also compares the spring back phenomenon with analytical results.

In [2] author evaluates the maximum force acting on the rollers during the rolling process, for designing the rolling machine. In this study mathematical model for a force prediction on the rollers has been developed by considering coefficient of friction,material properties and geometrical parameters. It has been concluded that the proposed model can be effectively used to get roller bending force for the given geometrical parameters and material properties.

In [3] author presents case study and stress analysis of three roller bending machine .For getting parametric specifications of three-point bar bending machine, it has required the analysis by FEM & suitable software like ANSYS-V13, LS-DYNA, ABAQUS etc. It requires the gear train mechanism with the motor to drive the motor to transfer torque to overcome vertical load acting during operation and speed reduction for case bending and rolling process. So an analysis of the load acting on the gear tooth is necessary to set a minimum inside radius of rollers of a roll of the 3 roller bending machine. The various theories have been developed on bending these are described below.

## 3 THEORIES REGARDING BEAM BENDING

### 3.1 Quasistatic bending of beams [4]

A beam deforms and stresses develop inside it when a transverse load is applied to the beam. In the quasistatic bending,

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assume the amount of bending deflection and the stresses that developed not to change w.r.t time. When a horizontal beam supported at the ends and loaded downwards in the middle, the material on the upper side of the beam is compressed while the material at the lower side is stretched. Due to these two forms of internal stresses produced by lateral loads, one is shear stress parallel to the lateral loading plus complementary shear stress on planes perpendicular to the load direction and second direct compressive stress in the upper region of the beam as well as direct tensile stress in the lower region of the beam.

These last two forces form a couple or moment as they are equal in magnitude and opposite in direction. This bending moment resists the deformation characteristic of a beam. The stress distribution in a beam can be predicted quite accurately even when some simplifying assumptions are used.

### 3.2 Euler-Bernoulli bending theory

Element of a bent beam the fibers form arcs, the top fibers get compressed at the same time bottom fibers stretched. In the Euler-Bernoulli theory of slender beams, a major assumption is 'plane sections remain plane'. It means no any deformation due to shear. Also, this linear distribution is only applicable if the maximum stress is less than the yield stress of the material. The material stresses that exceed yield refer to plastic bending. At yield point, so the maximum stress experienced in the section (at the farthest points from the neutral axis of the beam) is defined as the flexural strength.

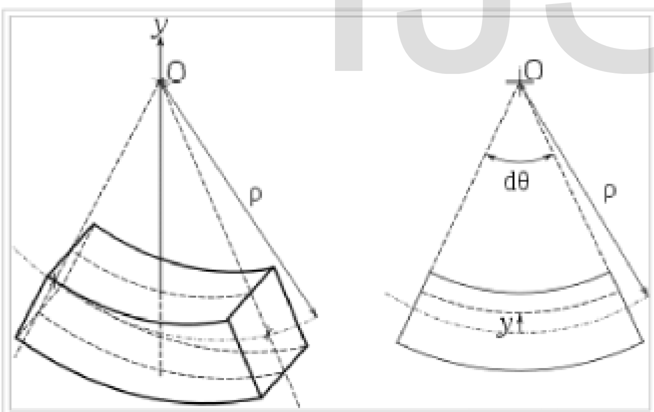


Fig 1. Beam showing deformation

The Euler-Bernoulli equation for the quasistatic bending of slender, isotropic, homogeneous beams of constant cross-section under an applied transverse load q(x) is shown below. The Isotropy is uniformity in all orientations.[5]

$$E I \; d^4w(x) / dx^4 = q(x)$$

Where ,E is the Young's modulus, I is the area moment of inertia of the cross-section, and W(x) is the deflection of the neutral axis of the beam.After a solution for the displacement of the beam has been obtained, the bending moment (M) and shear force (Q) in the beam can be calculated using the relations as below.

$$M(x) = -EI \; d^2w(x) / dx^2$$

$$Q(x) = dm / dx$$

Simple beam bending is often analyzed with the Euler-Bernoulli beam equation. The conditions for using simple bending theory are:

When the beam is subject to pure bending that is the shear force is zero, and no torsional or axial loads are present. The material is isotropic and homogeneous. The material obeys Hooke's law (it is linearly elastic and will not deform plastically). The beam is initially straight with a cross section that is constant throughout the beam length. The beam has an axis of symmetry in the plane of bending. The proportions of the beam are such as it would fail by bending rather than by crushing, Wrinkling or sideways buckling. Cross-sections of the beam remain plane during bending.

A beam is deflected symmetrically Compressive and tensile forces developed in the direction of the beam axis under bending loads. Compressive and tensile forces induce stresses on the beam. The maximum compressive stress is found at the uppermost edge of the beam while the maximum tensile stress is located at the lower edge of the beam. As the stresses between these two opposing maxima vary linearly, there exists a point on the linear path between them where there is no bending stress. The position of these points is on the neutral axis, due to this develop an area with no stress and the adjacent areas with low stress. Using the uniform cross section beams bending is not a particularly efficient means of supporting a load as it does not use the full capacity of the beam until it is on the brink of collapse. Wide-flange beams (I-beams) and truss girders effectively address this inefficiency as they minimize the amount of material in this under-stressed region.

The bending stress can be determined by classic formula :

$$\sigma = M y / I$$

Where, $\sigma$ - The bending stress , M - The moment about the neutral axis , y - The perpendicular distance to the neutral axis , I - The second moment of area about the neutral axis x. This classical equation gets extended by considering plastic bending, large bending, unsymmetrical bending.

### 3.3 Extensions of Euler-Bernoulli beam bending theory for plastic bending

The equation $\sigma = M y / I$ is valid only when the stress at the extreme fiber (i.e. the portion of the beam farthest from the neutral axis) is below the yield stress of the material from which it is constructed. During higher loadings the stress distribution becomes non-linear so ductile materials will gradually enter a plastic hinge. It states the magnitude of the stress is equal to the yield stress everywhere in the beam. At the neutral axis the stress changes from tensile to compressive. This plastic hinge state is typically used as a limit state in the design of steel structures. It means we should be considering the Elasto-plastic properties of materials. Where the Elastoplasticity [6] is State of a substance subjected to a stress greater than its elastic limit but not so great as to cause failure, in which it exhibits both elastic and plastic properties.In the Euler -Bernoulli Equation we get the equation for bending moment as

$$M = \sigma_b \, w \, t^2 / 6 \quad \ldots\ldots\ldots\ldots(1)$$

What will happen if the load is increased? The layers will be additionally elongated by an amount that is proportional to the distance y of the layer from the zero line. But the deformation of the layers most distant from the zero line will grow without an increase in stress beyond the yield point. So the Fig 2 a show the situation in which deformation is twice as compared to Fig 2.b. In this case, the yielding area begins at y = h/4 and extends to y = h/2. The equilibrium equation for this case is given below. [7]

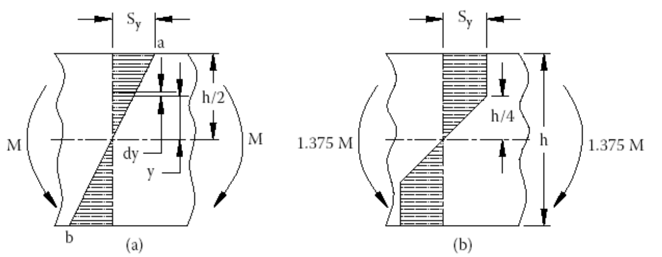


Fig 2.Stress distribution diagram for elastic and plastic bending. [7]

As we can see, bending moment M* is 37.5% greater than M:

M* / M =11x6/48 =1.375……..(2)

This load capacity is calculated for regtanguler section.
In the derive the equation of moment for elastic and perfectly plastic material in the book Mechanics of Sheet Metal Forming . The relation of moment and curvature for for elastic and plastic is given in fig3.
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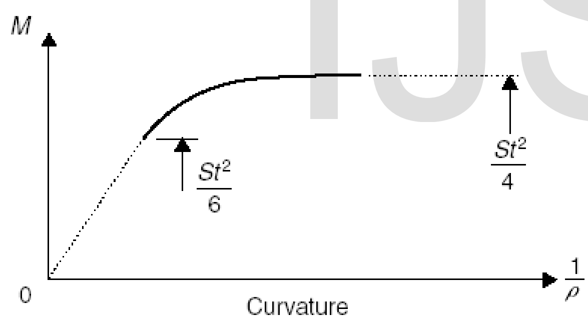



Fig 3. The relation between moment and curvature for elastic and plastic material.[8]

According to [9] $2/\surd 3\, б_f = S$
Where ,S= Plain strain yield stress for elastic bending and flow stress for plastic bending . $б_f$ =flow stress which means the stress at which material yield in somple tension.The limiting elastic moment is given by

$$M_e = St^2/6$$

If the curvature is greater then the limiting elastic moment reaches about five times before becoming constant, so for a rigid perfectly plastic model the moment $M_p$ is

$$M_p = St^2/4$$

Therefore $M_p = 1.5\, M_e$ ………………………………(3)

### 3.4 Complex or asymmetrical bending

The equation of Euller is only valid if the cross-section is symmetrical. If there are homogeneous beams with asymmetrical sections, then axial stress in the beam is given by

$$б_{z(y,z)} = - [( M_z I_z + M_y I_{yz} ) y / ( I_y I_z - I^2_{yz}) ] + [( M_y I_z + M_z I_{yz} ) z / ( I_y I_z - I^2_{yz}) ]$$

where y, z are the coordinates of a point on the cross section at which the stress is to be determined as shown in fig 4 above, $M_y$ and $M_z$ are the bending moments about the y and z centroid

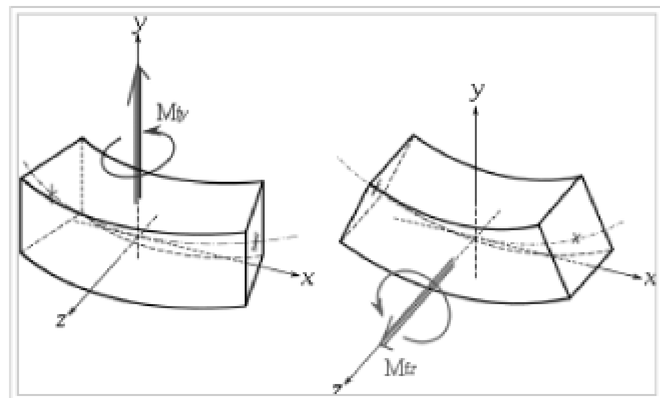


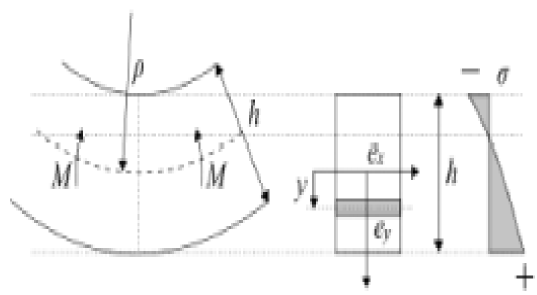

Fig 4. Moments at different directions

axes, $I_y$ and $I_z$ are the second moments of area (distinct from moments of inertia) about the y and z axes, and $I_{yz}$ is the product of moments of area. The bending stress at any point on the beam cross section regardless of moment orientation or cross-sectional shape can be calculated by using above equation. Assume that $M_y$ , $M_z$, $I_{yz}$ ,$I_z$ ,$I_y$ do not change from one point to another on the cross section.

### 3.5 Large bending deformation

Nonlinearities exist in an equation of motion when the products of variables, or their derivatives, exist. They can also exist when there are discontinuities or jumps in the system. There are several sources of nonlinear behavior.Geometric nonlinearity is the major source in bending. This characteristic is important for large deformations, also the systems that may fail due to the buckling. In beams and plates, the nonlinearity is from the nonlinear strain equations, where the transverse displacement is coupled to the axial strains. As a result, the mid-plane stretching of the beam or plate may occur. Nonlinear moment-curvature relationship becomes significant when we consider large deformations without stretching. This analysis does not consider the slope of the deflected middle surface to be small compared to unity. This analysis is usually done in terms of the slope of the beam.
Another cause of nonlinearity is material properties. These nonlinearities would render Hooke's law invalid because Hooke's law is a linear relationship between stress and strain. Hooke's law would have been altered in order to account relationship for the nonlinear behaviour. We can define the slope of the linear region in the elastic region of materials, as the Young's modulus. It is necessory to understand the system in terms of the material model, loading and expected response, in order to determine where a linear approximation is adequate and where the use of a nonlinear theory is needed. When a thin elastic plate undergoing small deformations, i.e. w < 0.1h ( where w is the transverse deflection

and h is the plate thickness) it is reasonable to ignore geometric nonlinearities and use linear plate theory. However in larger deflections the middle surface of the plate begins to stretch or the in-plane motion of the plate edges become significant. When these effects become important one needs to consider geometrically nonlinear plate theory, which was first derived by von K´arm´an in 1910[9]



. Fig 5. Shifting of neutral layer from neutral axis.

At large deflections, the material becomes plasticized and there is no longer any linear relation between deflection and force

For large deformations, the stress is calculated using an extended version of this formula. The following assumptions must be made:

Assumption - flat sections before and after deformation the considered section of body remains flat (i.e., is not swirled).Shear and normal stresses in this section that are perpendicular to the normal vector of cross section have no influence on normal stresses that are parallel to this section. Considerations for large bending should be implemented when the bending radius is smaller than ten section heights h:

$$\rho < 10\,h$$

For this case stress in large bending is calculated as:

$$\sigma = (F/A) + (M/\rho A) + (My/I'_x)\,[\,\rho/(\rho + y)\,]$$

where, F the normal force,A the section area ,M the bending moment ,ρ The local bending radius (the radius of bending at the current section) ,$I'_x$ the area moment of inertia along the x axis, at the place ,y the position along y axis on the section area in which the stress is calculated

When bending radius ρ approaches infinity and $y \ll \rho$ , the original formula is back:

$$\sigma = (F/A) + (M\,y/I) \quad \ldots\ldots\ldots..(4)$$

After Euller-Bernoullie the Timoshenko developed theory by considering shear factor and angle changes by shear.

### 3.6 Timoshenko bending theory

Deformation of a Timoshenko beam the normal rotates by an amount which is not equal to amount of moment.In 1921, Timoshenko improved upon the Euler-Bernoulli theory of beams by adding the effect of shear into the beam equation. In Euler – Bernoulli beam theory, shear deformations are neglected, and plane sections remain plane and normal to the longitudinal axis. In the Timoshenko beam theory, plane sections still remain plane but are no longer normal to the longitudinal axis. The difference between the normal to the longitudinal axis and the plane section rotation is due to the shear deformation.

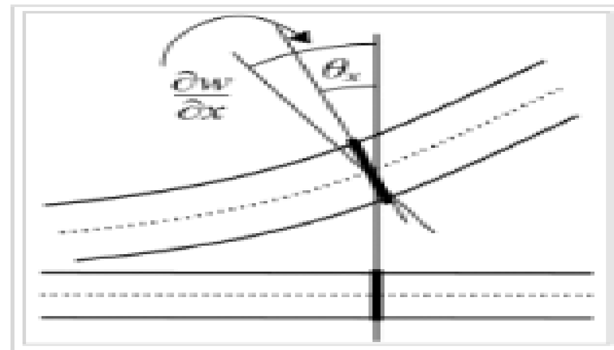


Fig 6. Position of plane during bending

In Euler-Bernoulli beams, transverse shear stress is not taken into account where as in Timoshenko beams transverse shear stresses are taken into account. The reason why transverse shear stress is not taken into account in Euler - Bernoulli beams the bending is assumed to behave in such a way that cross section normal to the neutral axis remain normal to the neutral axis after bending. Classical beams are very good for thin beam applications whereas Timoshenko beams well for thick beams. In case of Timoshenko beams initially cross section in normal to the neutral axis but does not remain normal after bending.[9]These relations are shown in figure 7 .There is one little difference - shear deformations that we added to Euler bending. More particularly we understand that we have a bending angle and deflection as deformations at beam bending process. But accuracy increases by adding a shear angle to bending angle. Then we have total angle instead of bending angle only.



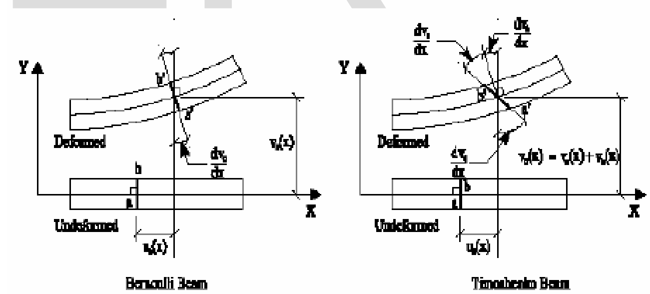



Fig 7. Difference between Bernaullies And Timoshenko beam deflection.[9]

The kinematic assumptions of the Timoshenko theory are, Plane normal to the axis of the beam remain straight after deformation.There is no change in beam thickness after deformation. However, normal to the axis are not required to remain perpendicular to the axis after deformation.The equation for the quasistatic bending of a linear elastic, isotropic, homogeneous beam of constant cross-section beam under these assumptions is

$$EI\ d^4w(x)/dx^4 = q(x) - (EI/kAG)/(d^2q/dx^2)$$

where ,I the area moment of inertia of the cross-section, A the cross-sectional area,G is the shear modulus, k shear correction factor. For materials with Poisson's ratios (υ) close to 0.3, the shear correction factor for a rectangular cross-section is approximately

$$k = (5 + 5\upsilon)/(6 + 5\upsilon)$$

The rotation ( $\varphi(x)$ ) of the normal is described by the equation.

$d\varphi / dx = -(d^2 w / dx^2) - (q(x) / kAG)$

The bending moment (M) and the shear force (Q) are given by

$M(x) = -EI\, d\varphi / dx$ ; $Q(x) = kAG\,((dw / dx) - \varphi)$

$= -EI\, d^2\varphi / dx^2 = dM / dx$

Tarsicio put an experimental results on this theory. In the paper Large and small deflection of a cantilever beam the Tarsicio Belendez et al. presents the result after doing the experiment on a cantilever beam to measure the deflection of linear elastic material under the action of an external vertical concentrated load at the free end in their laboratory. They analyses the experimental results with theory. During their study they found that in the text book on physics , mechanics and elementary strength of material the discussion is limited to the consideration of small deflections. The analysis of large deflection of elastic material and its solution is difficult. [10]

Now it should be apparent that, the factors influencing a bending project are extensive. Criteria for evaluation should be indicative of individual company philosophies and requirements. A well thought out, consistent and pragmatic approach will yield far better results than reactionary decision making.

To solve that problem they obtain differential equation for deflection curve in general case of large deflection as well as the equation that determine the Cartesian coordinates of each point on elastic curve .For making this they differentiate the following equation w.r.t. s $EI\, d\varphi / ds = M$

We get $EI\, d^2\varphi / ds^2 = dM / ds$

Where the value of M at a point A with Cartesian coordinates (x,y) as shown by fig8. is given by the equation $M(s) = (L - \delta x - x)$

And solve the equation for non-linear deferential equation . After conducting experiment they found the deflection due to its self weight and which is by calculation was not same. This difference has come due to lack of consideration of self weight of material into the theoretical formulae.

The Bernoulli-Euler elementary theory of bending (ETB) of beam disregards the effect of the shear deformation. The theory is suitable for slender beams and is not suitable for thick or deep beams since it is based on the assumption that the transverse normal to the neutral axis remains so during bending and after bending, implying that the transverse shear strain is zero.

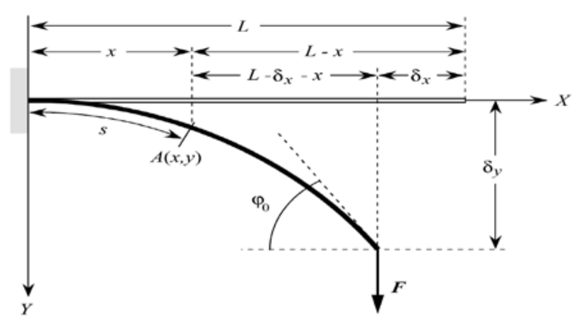



Fig 8.Deflection curve in general case of large deflection to determine the Cartesian coordinates. [10]

Since the theory neglects the transverse shear deformation, it underestimates deflections and overestimates the natural frequencies in case of thick beams, where shear deformation effects are significant.

The first order shear deformation theory (FSDT) of Timoshenko includes refined effects such as the rotatory inertia and shear deformation in the beam theory. Timoshenko showed that the effect of transverse shear is much greater than that of rotatory inertia on the response of transverse vibration of prismatic bars. In this theory transverse shear strain distribution is assumed to be constant through the beam thickness and thus requires shear correction factor to appropriately represent the strain energy of deformation. In this paper, consistent hyperbolic shear deformation theory previously developed by Ghugal and Sharma[11] for thick beams is used to obtain the general bending solutions for thick isotropic beams. The theory is applied to uniform isotropic solid beams of rectangular cross-section for static flexure with various boundary and loading conditions. They form the general formula for Cantilever beam with concentrated load P at free end as below

$w(L/2) = [(PL^3) / 48EI]\,[1 + 2.4(1 + \mu)h^2/L^2]$ …….(5)

Where, w(L/2) distance at mid of span where we get maximum bending, P concentrated load ,L span , h the total depth of beam, μ Poisson's ratio of the beam.The results are compared with those of elementary, refined and exact beam theories available in the literature to verify the credibility of the present shear deformation theory.[11]

From the book [ 12] the force required for bending is f

$F = KLSt^2 / w$ ………………….(6)

F is bending force required , K opening factor (1.33 for opening 8 times metal thickness,1.2 for opening 16 times metal thickness and 1.67 for U bending ),L width ,w distance between points,t thickness of material,S ultimate tensile strength.

The elastic deflection when loaded as simply supported the force required for deflection is given as

$F = 48EI\,\delta_c / L^3$ ……………………(7)

Where $\delta_c$ is deflection at mid of span.

## 4 CASE STUDY

M. K. Industry Plot No.'C' 2/3 , M.I.D.C Satara-415001 having the three roller simplisupported sheet bending machine.In this machine there are three rollers One at top and two at bottom.Drive is given to both bottom gears.The force is exerted on material by top roller. In this case study we are going to add machine specification and material properties in the above formulae and calculate the force. From this force, diameter of roller and revolution of roller calculate the power. This power is going to compare with actual power.

The list of formulae is

1. $M = \sigma_b\, wt^2/6$
2. $F = 1.37\sigma_b\, wt^2/6$
3. $M_e = 1.5M_p$
4. $\sigma = (F/A) + (My/I)$
5. $w(L/2) = [(PL^3) / 48EI]\,[1 + 2.4(1 + \mu)h^2/L^2]$
6. $F = KLSt^2 / w$
7. $F = 48EI\,\delta_c / L^3$

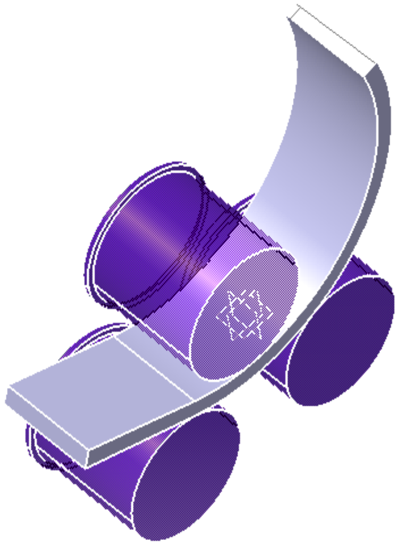


Fig 9.Construction of 3 roller bending machine using Catia

TABLE 1
MACHINE SPECIFICATION

| Specification Name | Value |
|---|---|
| Distance between bottom rollers | 300 mm |
| Diameter of bottom rollers | 150 mm |
| Speed of rotation | 3 rpm |
| Power of motor | 7.5 kW |

TABLE 2
MATERIAL SPECIFICATION

| Specification Name | Value |
|---|---|
| Material to be bend | ASTM a36 |
| Maximum diamention to be rolled | 1500 mm x 20 mm |
| Young's Modulus | 200GPa |
| Dencity | 7,800 kg/m3 |
| Poisson's ratio | 0.260 |
| shear modulus | 79.3 GPa |
| yield strength | 250 MPa |
| ultimate tensile strength | 400–550 MPa |
| Bending stress | 22000 Psi or 152MPa[13] |

Operation- Bend the material for radius 1000 mm. After bending we get the deflection in between rollers is 17.31mm

## 5 NUMERICAL RESULTS AND GRAPH

By utilizing above tables and reviewed formulae above the value of force and power are shown in the following table.

TABLE 3
NUMERICAL RESULTS

| Formula No. | Force Value get In kN | Power get in kW |
|---|---|---|
| 1 | 202.66 | 4.078 |
| 2 | 277.64 | 6.54 |
| 3 | 303.99 | 6.117 |
| 4 | 194.04 | 4.57 |
| 5 | 6075.6 | 143.15 |
| 6 | 1336 | 31.478 |
| 7 | 6155 | 145 |

The Fig 10 shows that variations in values in various formulae developed accordingly. The following figure shows that the graph of above table.From the graph we can say that first four formulae gives optimum value for power of this machine .The other three shows greater values which is not feasible.

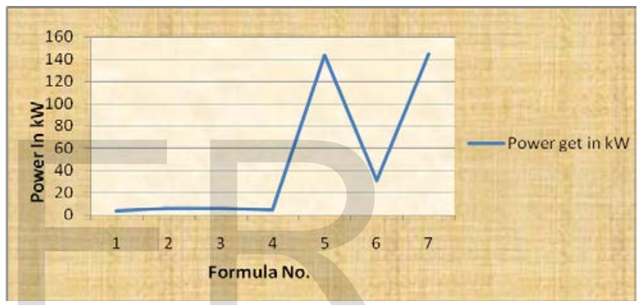



Fig 10. Graphical representation of power

## 6 CONCLUSION

Timoshenko has required differential equation which is difficult for those who don't know about mathematical formulation The first three formulae give realistic results for this application. But during practical approach there is impact of the number of passes. In all above formulae the number of passes is not considered. All the parameters those required for calculating force is not considered together.
From above discussion the conclusion comes out that , there should be added the remaining parameter into Euler Bernoulli's classical equation then it will give correct results. In the future we are going to derive the predicted formula with considering number of passés and rolling diameter for this application with experimentation.

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